A new methodology for evaluating the risk of gun jump during post-perforating surge is proposed and verified against field data. This methodology is backed up by an advanced mathematical model, which fully integrates transient flow in the formation and surge dynamics in the wellbore triggered by the creation of perforation tunnels under static underbalance.

The technique obtains the viscous drag on tool string and the gun string dynamics, which may induce cable tangling or breaking due to gun jump.

This model is used for the gun jump risk prediction and mitigation by evaluating available options in perforating job design, such as the shot density, the thickness of perforated interval and the height of liquid cushion. Field examples from Indonesia are utilized to demonstrate the usefulness of this new technique.
A Gun Jump Model for Underbalance Perforating
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Abstract
A new methodology for evaluating the risk of gun jump during post-perforating surge is proposed and tested against field data. This methodology is corroborated by an advanced mathematical model of transient flow into the wellbore triggered by the creation of perforation tunnels under static underbalance and fully coupled with evolving transient flow inside the wellbore containing the gun string. The model predicts the viscous drag on the tool string and the gun string dynamics, which may induce cable tangling or breaking owing to gun jump. This model is used to mitigate the risk of gun jump by varying perforating job design options such as shot density, thickness of the perforated interval, and height of the liquid cushion. Examples of model application to the analysis of field data from Indonesia are provided.

Introduction
There are two basic mechanisms of gun jump during perforating underbalanced.\(^1\) One is associated with the dynamic impact of the shock wave created after initiation of the shaped charges on the gun string inside the borehole. This shock wave provides the gun string with momentum that can force it to move. This mechanism of gun jump can be activated perforating underbalance or overbalance and may result in both upward and downward gun movement, which is usually mitigated to some extent by shock absorbers included in gun strings. This mechanism of gun jump is not considered in the current study.

The other mechanism of gun jump is related to the post-perforating surge from the reservoir into the wellbore through perforation tunnels. If the underbalance pressure is high, the transient reservoir response coupled with the dynamic level of liquid inside the wellbore may create fast wellbore fill-up accompanied by viscous drag on the gun string and cable, causing the gun string to move upward.\(^2\) The more powerful the surge, the higher the risk of gun jump owing to viscous drag on cable. The risk of gun jump can be mitigated by reducing the static pressure underbalance, shot density, and thickness of the perforated interval. All these measures, however, can compromise the quality of perforation cleanup and the targeted well productivity. A reliable and quantitative technique is therefore needed to optimize perforating design, in particular, to choose the optimum cushion level, which would mitigate the risk of gun jump while providing maximum cleanup efficiency.

Surge Flow Physics
The reservoir surge starts immediately after perforating with static pressure underbalance and continues until the equalization of pressure between the wellbore and the reservoir is achieved. The transient reservoir response during surge is coupled with the borehole fluid dynamics and controlled by different physical mechanisms during its evolution in time.

As soon as the perforation tunnels have been created as shown in Fig. 1, the reservoir is in hydraulic communication with the wellbore, which is filled by the completion fluid up to an appropriate level with an air cushion above it. The initial size of the liquid column inside the wellbore \(L_s\), or the size of the air cushion \(H_s\), is the main parameter that controls the static pressure underbalance during underbalanced perforating.

The pressure differential between the wellbore and the reservoir, which is equal to the static pressure underbalance, is usually established in a very short time after detonating shaped charges. The pressure waves triggered by the detonation...
propagate from the perforating gun along the wellbore upward and downward. They eventually dissipate as a result of multiple reflections and frictional energy losses. After that, one can assume that the reservoir responds to the applied static pressure underbalance. Due to its short duration, the initial dynamic reservoir response is neglected compared to the total duration of surge.

The sequence of events during reservoir surge with their characteristic times is shown in Fig. 2 and Fig. 3.

Phase 1 (left picture in Fig. 2) is triggered by the applied static pressure underbalance and controlled primarily by the wellbore fluid compressibility, because the initially stagnant wellbore fluid has to be accelerated to start flowing upward along the wellbore.

The mechanism of acceleration is based on the transmission of tractive forces to fluid particles by pressure waves propagating in fluid with sound velocity, \( c \), which is of the order of 1.3–1.5 km/s. The top of the liquid column inside the wellbore will not start moving upward before the pressure wave initiated downhole in the perforated interval reaches it. This phase of surge occurs without variation of the liquid column volume inside the wellbore. The reservoir fluid is just displacing the wellbore fluid upward by compressing it. The duration of this phase depends on the length of liquid column inside the wellbore \( L_w \). It can be estimated as \( L_w / c \) and it is of the order of 1 sec for a length of the liquid column inside the wellbore of 1–2 km.

Phase 2 of surge (central picture in Fig. 2) begins when the pressure wave reaches the top level of the completion fluid. At this time, the whole liquid column becomes mobilized and starts moving upward, filling the wellbore and compressing the air cushion. This phase continues until the liquid column fills the top part of the wellbore by compressing the air cushion and effectively shutting down inflow from the reservoir. The behavior of reservoir–wellbore system during this phase is similar to the surge flow period of a closed-chamber test. The duration of this phase depends on the rate of surge from reservoir and the volume and pressure of the air cushion. The wellbore fill-up lasts usually for a few tens of seconds.

The final Phase 3 of surge (right picture in Fig. 2) features pressure buildup in the reservoir around the wellbore under the slow movement of wellbore fluid and continuing compression of the liquid column and air cushion. This phase of surge is similar to the conventional pressure buildup test used for the determination of reservoir parameters. The duration of this phase depends on the total volume of fluid produced from the reservoir and can be of the order of a few minutes to tens of minutes.

Thus, three phases of reservoir surge are identified: 1) early dynamic reservoir response coupled with fluid compression in the liquid column, 2) wellbore fill-up, and 3) reservoir pressure buildup under almost hydrostatic pressure distribution inside the wellbore.

Drillstem test and closed-chamber test theory can be applied to model the two last phases of post-perforating reservoir surge. The first phase of reservoir response, however, requires a different modeling approach. It is controlled by the compressibility of wellbore fluid coupled with the pressure diffusivity of reservoir. The duration of this phase is of the order of 1–2 sec. A fast pressure gauge is therefore needed for the acquisition of bottomhole pressure data.

![Fig. 1. Surge flow: problem geometry schematic.](image1)

![Fig. 2. Three phases of reservoir surge.](image2)

![Fig. 3. Sequence of events and typical timing of reservoir surge.](image3)
Reservoir Surge Simulation

The model of post-perforating surge presented here deals with the transient flow from the reservoir triggered by the pressure underbalance established at the sandface of the perforated interval and coupled with the wellbore fluid hydraulics. We neglect the shocks induced by the detonation of the shaped charges and assume that the drawdown pressure is equal to the static pressure underbalance during perforating. This drawdown pressure is applied instantaneously at the time \( t = 0 \) when the pressure distribution inside the wellbore is hydrostatic. We also assume for simplicity that the thickness of the perforated wellbore interval \( \delta \) is small compared with the length of the liquid column inside the wellbore and therefore its volume can be neglected. The perforated completion is represented as an openhole interval with borehole radius \( r_w \) and skin \( s \), which depends on the reservoir fluid mobility, shot density, geometry of the perforation tunnels, and cleanup efficiency (see Fig. 1). We assume in the forward model presented that the skin \( s \) is known either as a number or a function of time.

Reservoir Flow Equations

The transient fluid flow in reservoir is governed by the pressure diffusivity equation with the following initial and boundary conditions:

\[
\frac{1}{\eta} \frac{\partial p}{\partial t} = \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right), \quad \eta = \frac{k}{\rho \mu^*}, \quad r_w \leq r < \infty, \ldots \quad (1)
\]

\[p = p_\infty, \quad t = 0, \quad r \geq r_w, \ldots \quad (2)
\]

\[p(r, t) = \left( p - s \cdot \frac{\partial p}{\partial r} \right) \bigg|_{r=r_w} = p_s(t) \ldots \quad (3)
\]

\[p(r, t) = p_\infty, \quad r \rightarrow \infty \ldots \quad (4)
\]

The wellbore pressure opposite the producing interval or the bottomhole pressure \( p_w(t) \) is unknown in advance and has to be determined from the coupled reservoir–wellbore flow model.

We assume that the wellbore radius \( r_w \) is equal to the radius of its openhole and also that all parameters involved in the definition of pressure diffusivity are known or estimated.

The skin factor \( s \) for the wellbore with a perforated completion is an integral parameter characterizing productivity. It is defined with respect to the corresponding openhole wellbore and is affected by the combination of permeability damage during drilling and perforating (perforation length, diameter, shot density, and cleanup).

Eq. (3) means that there is a pressure drop at the wellbore wall across an ultrathin cylindrical layer adjacent to it, which is proportional to the pressure gradient or the flow rate in the reservoir just outside this layer. Although such a model of the productivity of a perforated completion hides all the details of reservoir flow to the perforations, it allows for significantly simplifying the mathematical formulation of the forward problem. Furthermore, the details of flow through the perforations are rarely available, making the calibration of a more comprehensive model of flow to the perforated completion impossible.

The perforated completion with known skin also can be represented by a wellbore with an equivalent radius \( r' \), which can be expressed through the actual radius and skin as

\[r' = r_w e^{-s} \ldots \quad (5)
\]

The relationship of Eq. (5) provides an important flexibility for wellbore representation in the model, especially for negative skin values when the boundary condition of Eq. (3) cannot be satisfied without modification of the wellbore radius.

In contrast to classic well testing theory, which is based on the assumption that skin is constant, the skin of a perforated completion varies with time during surge because of perforation cleanup. The determination of skin \( s \) from bottomhole pressure measurements during perforating represents the main challenge of surge data inversion and interpretation.\(^7\)

Flow Inside the Wellbore

The wellbore fluid hydraulics in its simplest version can be described by classic pipe flow theory. All the parameters involved in the model (pressure \( p \), density \( \rho \), and velocity \( w \)) are averaged over the cross-sectional area of the wellbore with the tool string and wireline cable inside, \( A(z) \). If heat exchange and temperature effects are neglected, the wellbore flow is governed by the following equations:

\[
\frac{\partial}{\partial t} (Ap_r) + \frac{\partial}{\partial z} (Ap_sw) = Q \ldots \quad (6)
\]

\[
\frac{\partial}{\partial t} (Ap_sw) + \frac{\partial}{\partial z} \left( C \cdot Ap_sw' \right) = -A \frac{\partial p}{\partial z} - F \cdot Ap_sw \ldots \quad (7)
\]

\[
\rho_i = F_i(p, C) \ldots \quad (8)
\]

Eqs. (6) and (7) express the continuity of fluid mass and momentum for flow along the wellbore. Eq. (8) is the equation of state (EOS) for the wellbore fluid, with the parameter \( C \) characterizing its composition.

The source term \( Q \) represents the influx from the reservoir into the wellbore per unit of wellbore length. It is equal to zero everywhere outside the perforated interval. Inside the perforated interval, the source term can be expressed through the reservoir pressure gradient at the wellbore as

\[
Q = \frac{2 \pi r_w k \partial p}{\mu \partial r} \ldots \quad (9)
\]

The parameter \( C_i \) is equal to 4/3 for a laminar flow and 1 for a developed turbulent flow, \( F \) is the frictional force per unit of wellbore length, and \( g \) is the gravity.
In pipe flow theory, the frictional force \( F \) is usually expressed through the average flow velocity \( w \) and the friction factor \( f \):

\[
F = \left(\pi/8\right) D \rho w^2,
\]

where \( D \) is the hydraulic (or equivalent) diameter of the wellbore with the tool string inside.

The friction factor for laminar flow, which is determined by the Reynolds number \( Re < 2000 \), is given by a classic formula:

\[
f = 64/Re, \quad Re = \rho w D/\mu,
\]

which involves the pipe wall roughness \( \delta \) and requires iterations of both \( Re \) and \( f \) for solving.

Transitional flow is observed at \( 2000 < Re < 4000 \) for smooth pipe. It represents a great uncertainty for modeling because the flow is usually unstable and the friction factor varies with time. To cover the full range of \( Re \) values in the simulations, we approximate the friction factor by a smooth function with monotone variation between its predicted values for laminar flow at \( Re < 2000 \) and for turbulent flow at \( Re > 4000 \). Our estimates indicate that the wellbore flow during the initial phase of surge is usually turbulent and therefore the correlation of Eq. (12) can be used. During the next two phases of surge, the flow has to slow down and, eventually, the laminar regime is established. How important this flow regime transition may be for surge modeling and analysis has yet to be investigated.

### Initial and Boundary Conditions for the Coupled Model

We neglect the perturbations inside the wellbore induced by the detonation gas expulsion and gun volume filling after initiation of shaped charges. The characteristic time of these processes is of the order of a few tens of milliseconds, which is significantly shorter than the time of liquid column pressurization, which is of the order of 1–2 sec. Thus, the wellbore fluid is considered stagnant at the beginning of surge

\[
w(z,0) = 0, \quad 0 < z < L_0,
\]

where \( L_0 \) is the initial height of the liquid column.

The initial pressure and density inside the wellbore at \( t = 0 \) are hydrostatic. If the density variation can be neglected, the initial condition for the pressure is

\[
p_r(z,0) = p_{ao} + \rho_g (L_0 - z), \quad 0 < z < L_0,
\]

where \( p_r \) is the initial pressure inside the air cushion.

We assume that the boundary condition at the reservoir–wellbore interface is changing instantaneously after detonation of the shaped charges and that the static pressure underbalance \( p_r - p_{ao} \), where \( p_{ao} = \rho_g L_0 \), is applied as a drawdown pressure within the perforated interval at \( t = 0 \):

\[
p_r(z,0) = p_{ao} = \rho_g L_0
\]

This pressure underbalance, \( p_r - p_{ao} \), is not maintained at \( t > 0 \) because the pressure within the perforated interval varies as a result of coupling reservoir flow and the flow of wellbore fluid. To model this flow interaction, one has to account for the mixing of reservoir fluid with wellbore fluid. This can be done by using mixing rules for the density of the mixture and by tracing the displacement of wellbore fluid by produced reservoir fluid. One can specify, for example, the initial controlled volume of the wellbore containing the perforated interval. The boundaries of this volume vary with the production of reservoir fluid. The integral continuity equation can be used for tracing the boundaries of the controlled volume, and the mixing rules are applied only to the fluids inside this controlled volume.

Additional boundary conditions have to be specified at the fluid displacement interfaces. They involve the pressure and velocity continuity. The velocity continuity can be simplified if the compressibility of fluid inside the controlled volume and rathole is neglected. In this case, only the top boundary of the controlled volume \( z_c(t) \) varies with time. Its coordinate has to satisfy the following equations:

\[
A \frac{dz_c}{dt} = \frac{2\pi rk_{c}}{\mu} \left( \frac{\partial p}{\partial r} \right)_{z_c}, \quad w(z_c,t) = \frac{dz_c}{dt},
\]

where \( h < L_0 \) is the thickness of perforated interval.

### Equation of State

The EOS for wellbore fluid in Eq. (8) is required for the closure of Eqs. (6) and (7), which involve three unknowns: \( p_r \), \( \rho_r \), and \( w \). An EOS is usually obtained from experimental data by interpolation or approximation around reference values of pressure and density. For example, if \( K_\rho \) is the bulk modulus of wellbore fluid at pressure \( p_{ao} \) and density \( \rho_{ao} \), then one has the following approximation for the density at neighboring pressure values:

\[
dp_r/\rho_r = dp_r/K_\rho
\]
Instead of bulk modulus $K$, the inverse quantity is often used, which is known as the compressibility of the fluid:

$$c_i = K_i^{-1} = \rho_i^{-1} \frac{dp_i}{dp}.$$  

(18)

Integrating Eq. (17), the EOS for the wellbore fluid can be represented as

$$\rho_i(p_i) = \rho_{i_0} \exp \left[ K_i^{-1} (p_i - p_{i_0}) \right].$$  

(19)

The variation of the bulk modulus $K_i$ with pressure also can be taken into account, using, for example, a general exponential relationship:

$$K_i(p) = K_{i_0} \exp \left[ \alpha_i (p - p_{i_0}) \right].$$  

(20)

The density then becomes a function of two parameters, $K_{i_0}$ and $\alpha_i$, which have to be chosen by fitting Eq. (20) to available experimental data. Substituting Eq. (20) into Eq. (17), one has after integration

$$\rho_i(p_i) = \rho_{i_0} \exp \left\{ \frac{1 - \exp[-\alpha_i (p_i - p_{i_0})]}{\alpha_i K_{i_0}} \right\}.$$  

(21)

The parameters $\alpha_i$ and $K_{i_0}$ of the EOS may depend also on the fluid composition:

$$\alpha_i = \alpha_i(C), \quad K_{i_0} = K_{i_0}(C).$$  

(22)

**Wellbore Pressurization Phase**

The initial phase of reservoir surge coupled with borehole fluid dynamics represents an unusual boundary condition for the pressure diffusivity Eq. (1). Indeed, the static pressure underbalance applied at $t = 0$ is not maintained at $t > 0$ because of the wellbore fluid compressibility. The pressure wave propagating upward inside the wellbore and compressing the completion fluid, which starts immediately at $t = 0$, increases the bottomhole pressure opposite the perforated interval, affecting the reservoir surge.

For classic reservoir flow theory, the flow rate $q(t)$, corresponding to instantaneously applied constant drawdown pressure, has a weak singularity at $t = 0$:

$$q(t) \propto t^{1/2}, \quad t \to 0.$$  

(23)

To investigate flow rate behavior for the coupled problem, we simplify the equations of borehole fluid dynamics. Let us assume that the cross-sectional area of the wellbore is constant, the flow regime is turbulent, with $C_f = 1$, and the friction force is a known function of fluid density and averaged velocity; i.e., $F = F(\rho_i, w)$.

Using the EOS of Eq. (17), Eqs. (6) and (7) can be rewritten in the form

$$\frac{\partial p_i}{\partial t} + K_i(p_i) \frac{\partial w_i}{\partial z} + w \frac{\partial p_i}{\partial z} = 0,$$

$$\frac{\partial w_i}{\partial t} + \frac{1}{\rho_i(p_i)} \frac{\partial p_i}{\partial z} = -g - \gamma \left( \rho_i, w \right).$$  

(24)

where $\gamma = F/\rho_i A$ is the normalized friction force.

This system of two hyperbolic equations has two sets of characteristics with appropriate relationships between dependant variables along them:

$$dz/\gamma = \lambda_2 = w \pm c,$$  

(25)

$$dw/\gamma = (cp_i)^{-1} dp_i/\gamma + g + \gamma = 0.$$  

(26)

Here $\lambda_1$ and $\lambda_2$ are the characteristic velocities and $c$ is the sound velocity in wellbore fluid defined as

$$c(p_i) = \sqrt{K_i(p_i)/\rho_i(p_i)}.$$  

(27)

There are two special cases when the relationships along the characteristics of Eq. (26) can be simplified.

If friction forces are neglected, $\gamma = 0$, and both density and sound velocity are constant, Eq. (26) becomes the exact differential equation, which can be represented after integration as

$$w - w^\prime = -g \left( t - t^\prime \right) \mp (cp_i)^{-1} (p_i - p_i^\prime).$$  

(28)

In the absence of both friction and gravity, $g = \gamma = 0$, and the relationships along the characteristics of Eq. (26) can be integrated without additional assumptions about density and sound velocity:

$$w - w^\prime = \mp \int_{p_i}^{p_i^\prime} \frac{dp}{c(p_i) \rho_i(p_i)}.$$  

(29)

The parameters with the superscript asterisk (*) correspond to a point somewhere on the characteristic of Eq. (25) used as a reference in integration.

Neglecting nonlinear effects associated with the pressure dependence of density and sound velocity, the relationship of Eq. (29) can be simplified to

$$w - w^\prime = \mp (cp_i)^{-1} (p_i - p_i^\prime).$$  

(30)

The operator “$\mp$” in Eqs. (28)-(30) corresponds to the characteristics $dz/\gamma = w \pm c$ with the opposite signs.
The schematic solution for the dynamic phase of wellbore fluid compression on the plane \((t, z)\) is shown in Fig. 4. It has two distinctive fronts: the fluid compression front \(CF\) propagating with the sound velocity and the displacement front \(DF\), which is the interface between the wellbore fluid and the reservoir fluid flowing into the wellbore. We assume for simplicity that the displacement is piston like, neglecting possible density contrast and fluid mixing.

The dashed lines represent two sets of characteristics given by Eq. (25). The infinitesimal perturbations in the flow region induced by boundary conditions and shock waves propagate along the characteristic trajectories satisfying Eq. (26). Strictly speaking, the trajectories of the characteristics are not straight lines as shown in Fig. 4 because both fluid density and sound velocity depend on pressure.

Comprehensive resolution of this solution requires solving the problem of flow inside the wellbore coupled with the reservoir influx. Leaving this study for the future, we consider next the approximate solution that can be obtained using the approach developed earlier for slug test and drillstem test analysis.3,4

**Lumped Model of the Dynamic Phase of Surge**

**Control Volume Concept**

The lumped model treats the fluid flow inside the wellbore as a moving liquid column imbedded in a control volume \(\Omega\) surrounded by a control surface \(S\). The laws of conservation for fluid mass and momentum are applied to this entire volume.5,9 The mass and momentum continuity equations for a moving control volume are

\[
\frac{d}{dt} \int_{S(\tau)} \rho d\Omega + \int_{S(t)} \rho (\vec{v} - \vec{u}) \cdot d\vec{S} = 0 \quad \text{.......................... (31)}
\]

\[
\frac{d}{dt} \int_{S(\tau)} \rho \vec{v} d\Omega + \int_{S(t)} \rho \vec{v} (\vec{v} - \vec{u}) \cdot d\vec{S} = 0 \quad \text{.......................... (32)}
\]

Here, \(\vec{u}\) is the local fluid velocity, \(\vec{U}\) is the velocity of the control surface, \(\tau\) is the viscous stress tensor, and \(\vec{F}\) is the external volumetric force.

Let us choose the control volume equal to the volume of mobilized fluid inside the wellbore as shown in Fig. 5. This control volume is bounded from the top by the compression front \(CF\) propagating upward with the sound velocity. It is convenient to choose its bottom boundary just above the gun string, treating the volume below as a storage volume with appropriate compressibility and additional hydraulic resistance for the fluid flow from the reservoir into the control volume of the wellbore. The gun string is usually much shorter than the height of liquid column inside the wellbore during perforating. We also assume that the rathole is shallow compared with the wellbore depth and the height of the liquid column inside it. The stagnant fluid above the control volume separates it from the air cushion.

The origin of vertical axis \(z = 0\) in Fig. 5 was shifted to the level of the bottom boundary of the control volume inside the wellbore. For simplicity, the cross-sectional area of wellbore \(A\) is considered constant everywhere above the gun string.

**Normalized Pressure**

The initial pressure inside the wellbore is hydrostatic:

\[
p = p_s(z) = p_s + \rho_s g (L_s - z), \quad \text{.......................... (33)}
\]

It is convenient to subtract the hydrostatic component of pressure in Eq. (33) and to deal further with the pressure perturbation with respect to the initial hydrostatic pressure:
\[ \phi(z) = p - p_0 = p - p_s - \rho g (L_0 - z) \] \hspace{1cm} (34)

At the beginning of surge, the normalized pressure \( \phi \) is equal to zero along the liquid column.

For underbalance perforating, the hydrostatic bottomhole pressure \( p_b(0) = p_a + \rho g L_a \) is smaller than the initial reservoir pressure, which can be represented for similarity with Eq. (33) through the virtual height of the liquid column inside the wellbore \( L_1 > L_0 \):

\[ p_a = p_b + \rho g L_a \] \hspace{1cm} (35)

**Continuity Equation**

Let us apply the conservation laws of Eqs. (31) and (32) to the specified control volume of fluid inside the wellbore assuming that its top boundary is moving upward with the sound velocity:

\[ z_c(t) = c t \] \hspace{1cm} (36)

The continuity equation in this case requires that the production rate from reservoir \( Q \) should be equal to the fluid compression rate at the compression front \( z = z_c(t) \), which in turn is equal to the fluid flow velocity \( v \) behind the compression front; i.e., at \( z = z_c - 0 \). Because the fluid is stagnant ahead of the compression front (i.e., at \( z = z_c + 0 \)), the compression rate can be expressed through the pressure variation across the compression front \( p(z_c - 0) - p(z_c + 0) = \phi_c \) as

\[ \phi_c = \rho c v \] \hspace{1cm} (37)

This equation can be also represented in the form

\[ v = \left( \frac{\Delta p_c}{\rho_s} \right) c \] \hspace{1cm} (38)

due to the following relationships between the pressure and density variations, as well as the velocity of sound for linear elastic material:

\[ \frac{\Delta p_c}{\rho_s} = \frac{\Delta p}{K_s}, \quad c = \sqrt{\frac{K_s}{\rho_s}} \] \hspace{1cm} (39)

Thus, the continuity equation for the control volume of Eq. (37) correlates the surge rate

\[ Q(t) = v(t) A \] \hspace{1cm} (40)

with the pressure variation across the compression front

\[ \Delta p(t) = \phi_c(t) \] \hspace{1cm} (41)

The simple relationship of Eq. (37) has been obtained assuming uniform velocity along the mobilized liquid column. The deficiency of this assumption is obvious, because the pressure perturbations propagate along the liquid column with the finite velocity equal to the velocity of sound.

**Momentum Equation**

Using similar assumptions, we derive the following momentum conservation equation for the control volume from Eq. (32):

\[ \frac{d}{dt} \int_0^z \rho v A dz + \int_0^z \left[ \rho v (v - c) A \right]_{z_c} - \int_0^z \left[ \rho v' A \right]_{z_c} = \left[ \rho A \right]_{z = 0} - \left[ \rho A \right]_{z = z_c} - \Pi z \] \hspace{1cm} (42)

The term \( \Pi = 4A/D_p \) is the perimeter of the cross-sectional area of wellbore with a cable inside it and \( D_p \) is its hydraulic or equivalent diameter. Because the cable diameter is usually small compared with the wellbore diameter, its presence inside the wellbore has very small effect on the cross-sectional area, perimeter, and hydraulic resistance. At the same time, the calculation of the drag force on the cable requires caution, because the velocity profile around the cable, which determines the viscous stress at its surface, is not represented accurately by the average viscous stress \( \tau \) involved in Eq. (42).

If the effect of the cable on the hydraulic resistance of the wellbore is neglected, the viscous friction force in Eq. (42) can be expressed as

\[ \int_0^z \Pi z \] \hspace{1cm} (43)

where \( r_p \) is the inner radius of casing.

The resulting lifting force on the mobilized liquid column in Eq. (42) can be represented as

\[ \left[ \rho A \right]_{z = 0} - \left[ \rho A \right]_{z = z_c} = (\phi_w - \phi_c + \rho g z_c) A \] \hspace{1cm} (44)

where \( \phi_w \) is the perturbation of the bottomhole pressure with respect to the initial hydrostatic pressure at \( z = 0 \).

Using Eqs. (43) and (44), the momentum Eq. (42) can be simplified to the following:

\[ \dot{v} + \frac{f}{4r_p} v = \frac{\phi_w - \phi_c}{\rho z_c} \] \hspace{1cm} (45)

Here and as follows, the dot above a symbol means its derivative with respect to time.
Eq. (45) is similar to the Saldana-Ramey equation. Indeed, because \( v = \frac{Dv_z}{Dz_c} \), this equation rewritten with respect to the coordinate of the displacement front \( Dz \), becomes

\[
\ddot{Dz} + \frac{f}{4r_p} \big| \ddot{Dz} \big| = \frac{p_c - p_c}{\rho_c} - g, \quad z_c = ct \quad \text{………………(46)}
\]

Replacing the coordinate of the compression front \( Cz \) in the denominator with the coordinate of the displacement front \( Dz \) and the pressure at the compression front \( p_c \) with the pressure in the air cushion \( g \), one arrives exactly at the Saldana-Ramey equation:

\[
\ddot{Dz} + \frac{f}{4r_p} \big| \ddot{Dz} \big| = \frac{p_c - g}{\rho_c}, \quad z_c = ct \quad \text{………………(47)}
\]

**Lumped Model Formulation**

The lumped model thus involves three equations:

\[
\dot{v} + \frac{f}{4r_p} \big| v \big| = \frac{\rho_c - \rho_c}{\rho_c} - g, \quad z_c = ct, \quad \rho_c = \rho_c \quad \text{……(48)}
\]

with respect to four unknowns: \( v \), \( z_c \), \( \varphi_c \), and \( \varphi_w \). It can be reduced to a single equation for two unknowns, \( v \) and \( \varphi_w \), only:

\[
\dot{v} + \frac{f}{4r_p} \big| v \big| = \frac{\rho_c - \rho_c}{\rho_c} - g, \quad \text{……(49)}
\]

Model closure is achieved by coupling the wellbore flow with flow in the reservoir and requiring continuity of pressure and flux at the wellbore wall:

\[
p(r_w, t) = \varphi_a(t) + p_o + \rho_o \frac{L_o}{r_w}, \quad Q(t) = 2\pi r_w h u_w = vA \quad \text{………………(50)}
\]

Here, \( u_w \) is the virtual reservoir fluid velocity at the wellbore wall that corresponds to production through the perforated interval of wellbore. The reservoir pressure \( p(r, t) \) satisfies the pressure diffusivity Eq. (1) with the initial and boundary conditions of Eqs. (2)–(4), which involve the skin of perforated completion \( s \).

Eq. (49) requires the initial condition for the fluid velocity inside the wellbore \( v(0) = v_o \). It can be obtained from the fact that the initial pressure drop across the compression front \( \varphi_c(0) \) must equal the initial pressure underbalance \( p_r - p_w (-0) \) and therefore \( \rho_o g (L_o - L_s) = \rho_o v o \). This results in the following expression:

\[
v_s = \frac{g (L_o - L_s)}{c} \quad \text{………………(51)}
\]

For example, if the head loss \( \Delta L = L_o - L_s \) that is equivalent to the initial drawdown is equal to 1,000 m and the sound velocity in the completion fluid is about 1,300 m/s, the initial compression rate is \( v_o \approx 7.5 \text{ m/s} \). It is worth noting that this estimate of the initial flow velocity does not depend on reservoir performance, which means that the compression rate of Eq. (51) always has to be matched by the initial reservoir response.

Thus, the initial surge rate from reservoir is also finite:

\[
Q(0) = v_o A = \frac{\Delta L A}{c} \quad \text{………………(52)}
\]

This feature of the surge solution is different from the classic solution for production from the wellbore with constant drawdown, which has infinite initial production rate evolving as an inverse square root of time.

Obviously, under these circumstances, the initial bottomhole pressure \( p_w (+0) \) must be equal to the initial reservoir pressure \( p_r \), making the right-hand side of Eq. (49) equal to zero at \( t = 0 \).

**Storage Volume**

The storage volume, which was introduced for keeping the momentum equation for the lumped model simple, is supposed to be small compared with the volume of the liquid column inside the wellbore. The compressibility of the fluid in the storage volume results in slight delay in triggering displacement front propagation. Let us assume that the length of the gun string with all downhole tools is \( L_s \) and the depth of the rathole is \( L_s \). The storage volume then can be estimated for the gun string radius \( r_s \) as

\[
V_s = \pi r_o^2 L_o + \pi r_s^2 \left( r_o^2 - r_s^2 \right) \quad \text{………………(53)}
\]

For the initial drawdown pressure \( \Delta p \), the additional fluid volume \( \Delta V_s \) produced into the storage volume to achieve its initial compression matching the initial reservoir pressure is

\[
\Delta V_s = V \Delta p / K_o \quad \text{………………(54)}
\]

For the initial surge rate \( Q_o = v_o A \), the time delay \( \Delta t \) can be estimated as

\[
\Delta t = \frac{\Delta V_s}{Q_o} = \frac{V_s \Delta p}{v_o AK_o} \approx \frac{l_s \Delta p}{v_s K_i} \quad \text{………………(55)}
\]
where \( l_s \) is the length of storage volume. We neglected the gun string volume, assuming \( V_s / A \approx l_s \).

For \( v_o = 10 \text{ m/s}, \ l_s = 30 \text{ m}, \ \Delta p = 10 \text{ MPa}, \) and \( K_i = 1 \text{ GPa}, \) one obtains from Eq. (55) the estimate of the time delay \( \Delta t \approx 30 \text{ ms} \). It is indeed very small compared with the duration of the dynamic phase of surge, which is of the order of 1–3 sec.

The additional hydraulic resistance induced by the storage volume depends on its geometry and position with respect to the perforated interval of wellbore. It can be found prior to solving the lumped model of Eq. (49), for example, by simulating axisymmetrical steady-state flow inside the wellbore with the gun string inside. The velocity profile around the gun string can be used then for calculating drag force on the gun during surge. This simulation has to be conducted for the entire range of the surge rate, covering turbulent, transitional, and laminar flow regimes. The approximate solution can be obtained by neglecting the rathole contribution to the flow, but preserving its contribution to the compressibility of the storage volume and considering the flow in the annulus between casing and gun string with influx through part of the external boundary.

**Drag Force on Cable**

The liquid mobilized inside the wellbore during surge creates viscous drag on the cable that results in additional lifting force applied to the gun string. This lifting force can be expressed through the shear stress on the cable \( \tau_c \) as

\[
F_c(t) = 2\pi c \tau_c, \quad c = v, \quad \tau_c = \tau_c(v) \quad \text{……(56)}
\]

Because the length of the mobilized liquid column \( z_c \) is increasing with time and the displacement velocity \( v \) is probably decreasing with time, the viscous drag force \( F_c \) should have a maximum somewhere in the middle of the surge.

The shear stress on the cable can be obtained using the model of annular flow.\(^3\) It can be represented for both laminar and turbulent flow regimes as

\[
\tau_c = \frac{r_s}{2} \left( \frac{\rho_v v^2 f}{2D} \right) \left( a + \frac{1 - a^2}{2a \log a} \right), \quad D = 2\left( r_p - r_i \right), \quad a = r_i / r_p, \quad \text{……(57)}
\]

where \( D \) is the hydraulic diameter of annulus.

The friction factor \( f \) for the flow in annulus can be expressed through the Reynolds number:

\[
Re = \frac{\rho_v v D / \mu}, \quad \text{……(58)}
\]

For a laminar flow, \( Re < 2000 \), the friction factor is

\[
f = \frac{k_r}{Re}, \quad k_r = \frac{64}{r_p + r_i - \left( r_p - r_i \right)/\log \left( r_p/r_i \right)} \quad \text{……(59)}
\]

where \( k_r \) is the friction coefficient.

Substituting Eq. (59) into Eq. (57), one obtains the shear stress on the cable surface for a laminar flow in annulus:

\[
\tau_{cl} = -\frac{32\rho_v v^2 \left( 1 + a \right)}{2a \left( 1 + a^2 + b \right)}, \quad b = \frac{1 - a^2}{\log a} \quad \text{……(60)}
\]

For a turbulent flow, \( Re > 4000 \), the friction factor can be found from correlations based on experimental data. One can use, for example, the Colebrook-White correlation of Eq. (12) with the hydraulic diameter \( D \) corresponding to the annulus geometry.

Blevins\(^8\) also recommends the improved correlation for the friction factor based on a different definition of equivalent diameter \( D_e \) for turbulent flow in noncircular pipes and ducts proposed by Jones and Leung.\(^10\) This equivalent diameter \( D_e \) is expressed through the hydraulic diameter \( D \) and the friction coefficient for a laminar flow \( k_r \) as

\[
D_e = \frac{64D}{k_r} \quad \text{……(61)}
\]

and therefore the Colebrook-White correlation than can be rewritten as

\[
\frac{1}{\sqrt{f_r}} = -\log \left( \frac{e}{3.7} + \frac{2.51}{Re \sqrt{f_r}} \right), \quad e = \delta / D_e, \quad Re = \rho_v v D / \mu, \quad \text{……(62)}
\]

The shear stress on the cable surface for a turbulent flow is

\[
\tau_{ct} = -\frac{r_s}{2} \left( \frac{\rho_v v^2 f}{2D} \right) \left( a + \frac{1 - a^2}{2a \log a} \right) \quad \text{……(63)}
\]

The transitional flow regime, \( 2000 < Re < 4000 \), represents a great uncertainty for pipe flow applications because the friction factor is basically unpredictable by the steady-state theory owing to the non-steady-state flow nature. A simple way of dealing with this uncertainty is to interpolate the friction factor between its appropriate values for laminar and turbulent flow.

**Drag Force on Gun String**

The drag force on the gun string usually involves two components: form drag and skin friction. The form drag is created by nonuniform pressure distribution over the gun string surface. Because the redistribution of pressure around the gun string occurs very quickly, the form drag lasts short time. In this particular case, the time can be estimated as the time of storage volume pressurization. The skin friction, or
viscous drag, is determined by the shear stress at the gun string surface and is controlled by the fluid velocity in the annulus during surge.

The calculation of form drag on the gun string requires comprehensive modeling. Its maximum, however, can be roughly estimated through the drawdown pressure \( \Delta p \) and the cross-sectional area of gun string \( A_g \) as

\[
F_{\text{fr}} = \Delta p A_g \quad \text{(64)}
\]

For \( \Delta p = 10 \text{ MPa} \) and a 4.5-in. gun, we have \( A_g \approx 100 \text{ cm}^2 \) and \( F_{\text{fr}} \approx 10^4 \text{ kgf} \), or 22,000 lbf. This force is applied, however, to the gun string during the very short time it takes the initial compression wave to propagate along the gun string. If the gun length is of the order of 20 m and the velocity of sound is about 1,300 m/s, this time is about 15 ms. If the mass of the gun string is equal to (50 kg/m) \times (20 m) = 1,000 kg, the acceleration \( v_g \) that could be created by this force is about 10 m/s\(^2\); i.e., the order of the acceleration created by the gravity force \( g = 9.8 \text{ m/s}^2 \). This means that the estimated form drag has negligible effect on the gun string displacement during perforating.

The form drag increases with the gun size. If the resulting acceleration \( v - g \) is of the order of 10 m/s\(^2\), the gun jump velocity resulting from the form drag will be about 15 cm/s and the gun jump distance of the order of 1 mm; i.e., it is very small and can be neglected.

To estimate the viscous drag, let us assume that the fluid flow velocity in annulus between the gun string and the casing is about 20 m/s. The viscous drag force can be estimated as

\[
F_{\text{fr}} = 2\pi r_g L_g \tau \quad \tau = \frac{f_p \rho v^2}{8} \quad \text{(65)}
\]

Here, \( r_g \) and \( L_g \) are the respective radius and the length of the gun string. The friction factor \( f \) can be estimated from Eq. (12) for the Reynolds number \( Re = \rho_D v / \mu \) based on the appropriate hydraulic diameter of annulus \( D \).

For \( \rho = 1000 \text{ kg/m}^3 \), \( D = 5 \text{ cm} \), \( v = 10 \text{ m/s} \) and \( \mu = 1 \text{ cp} \), we obtain \( Re = 10^5 \) and the friction factor \( f \approx 0.01 \). The shear stress \( \tau \) in Eq. (65) is equal to 125 Pa.

Finally, the viscous drag on a gun string of radius \( r_g = 5 \text{ cm} \) and length \( L_g = 20 \text{ m} \) is about 750 N or 75 kgf.

Such a force is still small compared with the typical gun weight of this length, \((10 \text{ kgf/ft}) \times 60 \text{ ft} = 600 \text{ kgf} \), to trigger gun jump. This simple analysis confirms an intuitive conclusion shared by perforating experts that the predominant mechanism of gun jump during post-perforating surge has to be attributed to the viscous drag on cable.

Wellbore Fill-Up and Pressure Buildup Modeling

After the compression front hits the top of the liquid column inside the wellbore, the entire liquid column becomes mobilized and the wellbore fill-up phase of surge starts evolving as shown in Fig. 2. The simulation of this phase of surge can be conducted similarly to the simulation of the closed-chamber test or drillstem test. The only difference between the two cases is in the initial condition for the liquid inside the wellbore. In the drillstem test, the liquid column is usually short (a few tens of feet) and completely stagnant. The second phase of surge starts when the entire liquid column is already mobilized by the compression front propagation. Also, the initial height of the liquid column inside the wellbore during perforation can be very long (up to a few thousand feet).

This modification of initial conditions induces changes in the flow parameters for wellbore fill-up during post-perforating surge. The lumped model described in the previous section still can be used for the simulation of this phase of surge. It becomes identical to the Saldana-Ramey model applied to initially mobilized borehole fluid.

We use the Saldana-Ramey model with multiple enhancements to simulate the post-perforating surge. At this phase, we neglect all the details of borehole and gun string geometry, assuming that the perforated interval is small compared with the initial height of the liquid column inside the wellbore.

In the following section, we use a synthetic example to demonstrate the capabilities of the developed surge model.

Solution Analysis

Example input data used in the simulations is shown in Table 1. The static pressure underbalance during perforating is slightly above 2,000 psi.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation pressure</td>
<td>4,000</td>
<td>psi</td>
</tr>
<tr>
<td>Height of liquid column</td>
<td>4,000</td>
<td>ft</td>
</tr>
<tr>
<td>Height of air cushion</td>
<td>1,000</td>
<td>ft</td>
</tr>
<tr>
<td>Wellbore diameter</td>
<td>7</td>
<td>in</td>
</tr>
<tr>
<td>Pipe diameter (for through-tubing perforating)</td>
<td>3</td>
<td>in</td>
</tr>
<tr>
<td>Cable diameter</td>
<td>0.23</td>
<td>in</td>
</tr>
<tr>
<td>Thickness of perforated interval</td>
<td>30</td>
<td>ft</td>
</tr>
<tr>
<td>Formation permeability</td>
<td>270</td>
<td>md</td>
</tr>
<tr>
<td>Reservoir fluid viscosity</td>
<td>1.3</td>
<td>cp</td>
</tr>
<tr>
<td>Sound velocity in wellbore fluid</td>
<td>1300</td>
<td>m/s</td>
</tr>
<tr>
<td>Initial skin</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Final skin</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Time of skin variation</td>
<td>50</td>
<td>s</td>
</tr>
</tbody>
</table>

The results of modeling are presented in two series of figures, covering the first 3 sec (from Fig. 6 to Fig. 11) and the first 2 min (from Fig. 12 to Fig. 17) of surge. It is worth noting that the travel time for the compression front to the top of liquid column in this case is slightly less than 1 sec. The time of wellbore fill-up is about 70 sec. Thus, the time range from 0 to 120 sec allows for visualization of all three phases of surge.

The bottomhole pressure versus time is shown in Fig. 6 and Fig. 12. It decreases sharply from the initial reservoir pressure of 4,000 psi down to about 3,000 psi within first 200 msec and then increases gradually up to 3,100 psi until the compression front hits the top of liquid column inside the wellbore.
Form pressure = 4000 psi
Liquid column height = 4000 ft
Air cushion height = 1000 ft
Wellbore diameter = 7.00 in
Pipe diameter = 3.00 in
Cable diameter = 0.23 in
Formation thickness = 30.0 ft
Permeability = 270 md

Fig. 6. Bottomhole pressure during initial phase of surge.

Fig. 7. Production rate during initial phase of surge.

Fig. 8. Velocity of wellbore fluid displacement during initial phase of surge.

Fig. 9. Viscous drag on cable during initial phase of surge.

Fig. 10. Liquid column displacement during initial phase of surge.

Fig. 11. Mobilization front propagation inside the wellbore during initial phase of surge.
POST-PERFORATING SURGE DYNAMICS

Form pressure = 4000 psi
Liquid column height = 4000 ft
Air cushion height = 1000 ft
Wellbore diameter = 7.00 in
Pipe diameter = 3.00 in
Cable diameter = 0.23 in
Formation thickness = 30.0 ft
Permeability = 270 md

Fig. 12. Bottomhole pressure during post-perforating surge.

Fig. 13. Production rate during post-perforating surge.

Fig. 14. Velocity of wellbore fluid displacement during post-perforating surge.

Fig. 15. Viscous drag on cable during post-perforating surge.

Fig. 16. Liquid column displacement during post-perforating surge.

Fig. 17. Mobilization front propagation inside the wellbore during post-perforating surge.
By this time, the entire liquid column is mobilized and the pressure on its top boundary becomes equal to the pressure inside the air cushion. The change in boundary condition at the top of liquid column leads to further decrease in the bottomhole pressure, which continues for about 2–3 sec.

The bottomhole pressure reaches its minimum of about 2,500 psi and the starts increasing again with almost linear slope (see Fig. 12) until the top of liquid column reaches the top section of air cushion at the time of about 60 sec. The final phase of air compression is accomplished within 5 sec and the pressure buildup phase starts at the time of about 65 sec. The entire evolution of the bottomhole pressure is consistent with the basic physics of coupled flow inside the wellbore and reservoir.

The variations of all other parameters support this general observation. For example, the production rate shown in Fig. 7 and Fig. 13 is finite at the beginning of surge and decreases from about 13 kbb/day to 9 kbb/day within the first second of surge, when the compression front travels from the bottom to the top of liquid column. It starts increasing sharply after the entire liquid column is mobilized, reaching a slightly higher level of about 13.5 kbb/day, within the next second of wellbore fill-up. After reaching its maximum, the production rate decreases gradually to 10 kbb/day at end of wellbore fill-up phase (see Fig. 13) before it falls steeply to zero level at the time of 65 sec.

The variation of the displacement velocity (i.e., fluid flow velocity inside the wellbore) occurs similarly to the production rate as shown in Fig. 8 and Fig. 14. The displacement velocity is determined by the initial pressure underbalance and the rate of fluid compression inside the wellbore, which is controlled by the sound velocity. The displacement velocity is about 5 m/s at the beginning of surge. It decreases to 3.5 m/s by the end of the first second but starts increasing sharply after the entire liquid column is compressed and mobilized. The displacement rate hits its maximum of about 5.5 m/s in the next half of a second and then decreases gradually to about 4 m/s during wellbore fill-up. It falls steeply to zero within the next 5 sec.

The viscous drag on cable integrated along the cable length, which is exposed to viscous stress, is shown in Fig. 9 and Fig. 15. The drag is increasing with the mobilization of wellbore fluid by the propagating compression front. The cusp point at the end of the first second corresponds to switching from liquid column compression to wellbore fill-up. The drag force reaches 800 lbf at the end of first second and more than double in the next half of a second, reaching 1,900 lbf. Despite the continuing increase in the liquid column height during wellbore fill-up, the drag force on the cable is gradually decreasing owing to deceleration of the fluid moving upward. The drag force drops down to 1,400 lbf by the time of 55 sec before it falls to zero at the time of 65 sec. Thus, the drag force pulse shown in Fig. 15 in this particular case has an average amplitude 1,600 lbf and duration of 65 sec.

Despite the variation of fluid displacement velocity with time, the fluid displacement plots shown in Fig. 10 and Fig. 16 look almost linear as well as the displacement of the top of the liquid column in Fig. 11 and Fig. 17.

**Gun Jump Model and Risk Assessment**

Comprehensive modeling of gun string and cable displacement during surge is a difficult task if it targets all the details of gun jump phenomenon, which are well beyond the needs of this study. Indeed, regarding the risk of gun jump during perforating, one would probably like to know *Does it exist?* and if the answer is *Yes*, then *How high is it?*. Experience and common sense indicate that cable tangling and eventual breaking usually occur when the cable tension drops below some critical level. In accordance with field data, this critical level is of the order of 100 lbf, which is difficult to explain but easy to understand. This tension is much less than the typical weight of the entire conveyance string. It is measured at the wellhead and therefore can be different from the cable tension at the point of tool string attachment.

To model the reduction of cable tension during surge one has to consider the behavior of the entire conveyance string (gun string and cable) in the upstream flow of wellbore fluid mobilized by the powerful reservoir response and maintained until wellbore fill-up is completed. The drag force on the gun and cable pushes the entire conveyance string upward, eventually causing cable slackening.

Analysis indicates that, for deep wells, the weight of the gun string usually represents a small fraction of the cable weight. For example, the weight of typical fully loaded high shot density hollow-carrier gun is about 10 lbf/ft and for a gun-string of 20 ft, this gives the total weight of 200 lbf. The weight of the standard 0.23-in. wireline cable can be estimated as 100 lbf/kft. If the cable length is 15,000 ft, its total weight should be about 1,500 lbf.

Another observation is that the total viscous drag on the cable is usually much higher than the drag on the gun string. Because the length of the gun string is usually small compared with the cable length, the entire conveyance string can be approximated as an elastic stretchable cable with a mass at its end representing a gun string.

The equations governing vertical displacements of cable $u_c(z,t)$ and gun string $u_g(t)$ can be represented as

$$\frac{\partial}{\partial t}(mu_c) = \frac{\partial T}{\partial z} + P_c, \quad \dot{u}_c = \frac{\partial u_c}{\partial t}, \quad m = \frac{M_c}{L_c} \tag{66}$$

$$\frac{d}{dt}(M_cu_c) = P_c, \quad \dot{u}_c = \frac{du_c}{dt} \tag{67}$$

where $T(z,t)$ is the cable tension, $P_c(z,t)$ is the external force on cable per its unit length, $M_c$ is the mass of gun-string, and $P_c$ is the resultant force on it.

Let us assume that the entire conveyance string is in static equilibrium at the beginning of surge:

$$\frac{\partial T}{\partial z} + P_c = 0, \quad P_c = 0, \quad t = 0 \tag{68}$$

and also that all variables involved in Eqs. (66) and (67) (i.e., $u_c, u_g, T, P_c,$ and $P_c$) represent actually their deviations from the static equilibrium solution.
The coupling between Eqs. (66) and (67) is controlled by the law of dynamic interaction between the gun and the cable. A simple coupling can be based on the requirement of continuity of the entire conveyance string:

\[ u_c(t) = u_c(0, t) \]  \hspace{1cm} (69)

The model closure also requires a relationship between the cable tension \( T \) and its vertical displacement \( u_c \). Leaving comprehensive analysis of the interaction between the gun string and cable for the future, we derive a simple lumped model of gun jump during surge by treating the entire conveyance string as a single body.

Let us assume that the redistribution of tension along the cable occurs instantaneously and therefore can be expressed at any time through the gun displacement:

\[ T(z, t) = \frac{u_c(t)}{\lambda L_c} \]  \hspace{1cm} (70)

where \( \lambda \) is the cable stretch coefficient and \( L_c \) is the total cable length. The cable tension at the wellhead \( z = L_c \) therefore is

\[ T_c(t) = T(L_c, t) = -u_c(t) / (\lambda L_c) \]  \hspace{1cm} (71)

The external force on the cable \( P_c \) involves the drag force per unit length \( F_c/L_c \) and the reaction force from the gun. Using the Dirac delta function \( \delta(z) \) it can be represented as

\[ P_c = F_c / L_c + \delta(z) R \]  \hspace{1cm} (72)

The external force on the gun-string involves the drag on gun string \( F_c \) and the reaction force from cable \(-R\):

\[ P_g = F_g - R \]  \hspace{1cm} (73)

Substituting Eqs. (70) and (72) into Eq. (66), one obtains

\[ \frac{\partial}{\partial t} \left( \frac{m u_c}{\lambda L_c} \right) + \frac{u_c}{\lambda L_c} \frac{F_c}{L_c} = F_g + \delta(z) R \]  \hspace{1cm} (74)

The integration of this equation with respect to the vertical coordinate \( z \) from 0 to \( L_c \) gives

\[ M_c \ddot{u}_c + \frac{u_c}{\lambda L_c} = F_g + R \]  \hspace{1cm} (75)

Here \( U(t) \) is the average cable displacement, which has to be expressed through the gun string displacement \( u_c \). Because the cable tension \( T \) varies linearly with the vertical coordinate, the cable displacement \( u_c \) is also a linear function of \( z \):

\[ u_c(z, t) = u_c(t) \left( 1 - z / L_c \right) \]  \hspace{1cm} (76)

leading to the relationship

\[ U(t) = 0.5 u_c(t) \]  \hspace{1cm} (77)

Substituting Eq. (77) into Eq. (75) and adding the result together with Eq. (67) using Eq. (73), we obtain the lumped-type equation of conveyance string dynamics shown in Fig. 18:

\[ M_c \ddot{u}_c + \frac{u_c}{\lambda L_c} = F_g, \quad M = M_G + \frac{M_c}{2}, \quad F_g = F_c + F_c ... \]  \hspace{1cm} (78)

Due to the kinematics of the stretching cable, only one half of its mass is involved in the cumulative mass of the conveyance string. Eq. (78) is the equation of forced oscillation of the point mass \( M \) on the spring and it can be integrated for any arbitrary total drag force \( F_g(t) \):

\[ u_c(t) = c_1 \sinh \left( t - c_2 \right) + \beta \int_0^t F_g(\xi) \sinh \left( t - \xi \right) d\xi \]  \hspace{1cm} (79)

Choosing both parameters \( c_1 \) and \( c_2 \) equal to zero to satisfy the initial conditions \( u_c = u_c = 0 \), one arrives at the solution describing vertical gun string oscillation on cable induced by the cumulative lifting force.
Let us estimate the critical drag force for a 2 7/8-in. gun of length 20 ft on a 0.23-in. cable of length 5 km (or 15 kft).

The mass of a fully loaded high shot density gun at 6 spf and 60° phasing is 251 lbm, its volume is about 0.9 ft³, which is equivalent to 25 L and 55 lb mass correction for the Archimedean force, resulting in $\Delta M_g = 196$ lbm. The cable weight is about 100 lb/kft or 1,500 lbf for its entire length. The cable volume is equivalent to 134 L or 295 lbf of weight correction, leading to $\Delta M_c = 1,205$ lbm. Using Eq. (85), one obtains the critical drag in this case of the order 400 lbf.

The time of gun jump up to its maximum height can be estimated from Eq. (83). The stretch coefficient for the 0.23-in. cable is equal to $17 \times 10^{-2}$ ft/(lbf). The cumulative mass $M = (251 + 750)$ lbm = 1,000 lbm or about 454 kg. If the cable length is 5 km, the time of the first gun jump to its critical highest position is of the order 3 sec.

Because surge can continue for a few tens of seconds, the gun can be falling and jumping many times during surge unless cable tangling or breaking at some point interrupts its forced oscillation inside the wellbore. The cable tension at the lowest gun position is equal to $T_{cs} + F_g$, where $T_{cs} = (\Delta M_g + \Delta M_c)$ is the static tension at wellhead, and it should be less than the cable strength in its weakpoint.

The model created can help to design perforating jobs with the risk of gun jump and cable tangling or breaking under control.

### Field Examples

The data for two series of perforating jobs in Indonesia are shown below in **TABLE 2** and **TABLE 3**. The first table corresponds to perforating jobs run without any manifestation of gun jump and cable tangling (N series). The second table represents the data for jobs accompanied by gun jump (J series). The perforating environment was similar in both series: gas formations of a thickness of 3.5–7 m were perforated by 2 7/8-in. high shot density guns with a shot density of 6 spf and 60° phasing. A quick look at these tables reveals that the height of the liquid column was much longer in all Series N jobs except N6, providing better control over the post-perforating surge. The short liquid columns in Series J resulted in high static pressure underbalance and more powerful surge and evidently caused gun jumps above their critical levels.

**TABLE 2. NORMAL PERFORATING JOBS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>N6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation thickness, m</td>
<td>3.5</td>
<td>5.5</td>
<td>5.0</td>
<td>4.5</td>
<td>3.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Liquid column height, m</td>
<td>2,500</td>
<td>3,854</td>
<td>4,059</td>
<td>3,288</td>
<td>3,742</td>
<td>2,031</td>
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<tr>
<td>Air cushion height, m</td>
<td>1,169</td>
<td>174</td>
<td>1,200</td>
<td>2,000</td>
<td>717</td>
<td>3,744</td>
</tr>
<tr>
<td>Formation pressure, psi</td>
<td>4,468</td>
<td>5,443</td>
<td>6,023</td>
<td>5,813</td>
<td>6,475</td>
<td>5,478</td>
</tr>
<tr>
<td>Hydrostatic pressure, psi</td>
<td>3,626</td>
<td>5,590</td>
<td>5,887</td>
<td>4,769</td>
<td>5,427</td>
<td>2,946</td>
</tr>
<tr>
<td>Static underbalance, psi</td>
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<td>–147</td>
<td>136</td>
<td>1,044</td>
<td>1,048</td>
<td>2,532</td>
</tr>
</tbody>
</table>
TABLE 3. PERFORATING JOBS WITH GUN JUMP OBSERVED

<table>
<thead>
<tr>
<th>Parameter</th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
<th>J5</th>
<th>J6</th>
</tr>
</thead>
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<tr>
<td>Formation thickness, m</td>
<td>6.0</td>
<td>6.0</td>
<td>7.0</td>
<td>6.0</td>
<td>6.0</td>
<td>4.0</td>
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<td>636</td>
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<td>0</td>
<td>0</td>
<td>159</td>
<td>520</td>
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<tr>
<td>Air cushion height, m</td>
<td>3,944</td>
<td>4,600</td>
<td>4,700</td>
<td>4,600</td>
<td>4,395</td>
<td>2,429</td>
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<tr>
<td>Air cushion pressure, psi</td>
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<td>1,750</td>
<td>15</td>
<td>700</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Formation pressure, psi</td>
<td>3,870</td>
<td>5,057</td>
<td>5,046</td>
<td>4,519</td>
<td>7,478</td>
<td>3,254</td>
</tr>
<tr>
<td>Hydrostatic pressure, psi</td>
<td>937</td>
<td>1,750</td>
<td>15</td>
<td>700</td>
<td>631</td>
<td>1,254</td>
</tr>
<tr>
<td>Static underbalance, psi</td>
<td>2,933</td>
<td>3,307</td>
<td>5,031</td>
<td>3,819</td>
<td>6,847</td>
<td>2,000</td>
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</table>

The main unknown during surge simulation is the reservoir response controlled by the formation fluid mobility and the skin factor, which can vary significantly during surge. Under great uncertainty regarding these data, it was difficult to simulate all the cases accurately. For this reason, we illustrate below the gun jump model capability for a single normal job N1. The formation permeability in this case was 106 md and the porosity was 12.7%. The skin was estimated as 13.6 just after perforating and the final skin was 5 after 60 sec of surge. The drag force on the cable predicted by the model is shown in Fig. 19 for the liquid column height varying from 2000 m to 2500 m. The critical drag force of Eq. (85) is about 350 lbf, which is shown by the dashed line in Fig. 19. After the initial acceleration of the liquid column inside the wellbore, the drag on the cable is gradually increasing mainly due to the skin reduction until $t = 60$ sec. The drag on the cable starts gradually decreasing after the cleanup is finished.

The reduction of the initial liquid level down to 2300 m increases the drag on the cable up to about 400 lbf, i.e. above its critical level. Further reduction of $L_0$ down to 2000 m leads to dramatic increase in the drag force on the cable. For example, at $L_0 = 2000$ m, the predicted maximum drag is close to 900 lbf, i.e. almost 3 times above its critical level. The gun jump manifestation should be expected for the perforating under these conditions unless the shot density of the perforated interval is reduced. The integration of the gun jump model into the perforating job design should provide valuable means to assess and mitigate the risk of gun jump.

Conclusions

1. The forward model of post-perforating surge has been developed; the model covers three phases of surge: liquid column compression, wellbore fill-up, and pressure buildup.
2. This model is based on a 1D lumped simulation of pipe flow inside the wellbore coupled with 1D radial flow in the reservoir. The initial results of simulations are consistent with the basic physics involved in reservoir surge phenomenon.
3. In contrast to the classic transient solutions corresponding to production under instantaneously applied finite drawdown pressure, which features the production rate with inverse square root singularity, the solution obtained here has a finite production rate at the beginning of surge, which is controlled predominantly by the initial drawdown pressure, the compressibility of wellbore fluid, and the sound velocity.
4. The lumped gun jump model during a surge flow has been also developed. The gun displacement under some constraints is described by the equation of forced oscillation, which can be integrated for an arbitrary drag force on cable.
5. The model estimates the critical viscous drag force on a cable induced by a post-perforating surge, which can make cable slack. The critical drag on the cable may be very sensitive to the initial height of liquid column inside the wellbore, which controls the static pressure underbalance.
6. The drag force on the cable above the critical level creates a risk of cable tangling. The drag force below this critical level induces the oscillation of the gun string inside the wellbore. Cable safety under these circumstances can be compromised if the maximum cable tension induced at the lowest gun string position is larger than the cable strength at its weakpoint.

References


